Calculator for socially distant classrooms (and other spaces)

Required software

This tools is provided as a Mathematica notebook, and requires an installation of *Mathematica 12* to function. A CDF version of the tool also exists (useable on the free Wolfram Player) but it currently does not have the ability to overlay the calculated layouts with imported floor plans. The core calculation is otherwise the same, and produces layouts that can be used to arrange socially classrooms and other spaces in an optimal manner.

Scope

The calculator is designed to find an optimum seating arrangement in a conventional classroom or other space with an overall rectangular footprint. Each person is seated on a chair or at a desk that is assumed to be freely movable across the floor. One side of the room is designated as the front, where an instructor or presenter is located, or some other presentation or performance is taking place. The calculator attempts to fit as many seats as possible beyond a minimum distance from the front, placing the seats across the full width of the space in a pattern that ensures social distancing. Several different layouts are presented, making it possible to choose one that best avoids doors, pillars, lecterns etc., and that allows sufficient space for attendees to take their seats. This makes the calculator suitable not only for classrooms, but also for houses of worship, cinemas, theatres and other performance spaces. For other seating problems, such as the placement of tables with multiple seats, seating of a mixture of individual and family groupings, and seating in odd-shaped spaces, the calculator may offer some guidance, but there is no guarantee the seating arrangement will be optimal.

Principle of operation

(This section contains a mostly non-technical but somewhat lengthy summary of the theoretical foundation for the tool. Instructions for using the tool begin on page 5.)

Social distancing requires that each seated person maintains a minimum distance d (whether it be 1.5 m, 6 ft, or 2 m) to all others in the room. Replacing each seat with a disk of diameter d, we can see that this is equivalent to the well-understood problem of packing circular disks of diameter d into a confined space in such a way that no disks overlap. The density of disks is maximum when the disks (or seats) are arranged into a hexagonal grid, where every disk is in direct contact with six nearest neighbors (or equivalently, when every seat is spaced an equal amount from each of six nearest seats). Without considering the effect of the room shape, this means that one should be able to fit approximately 15% more seats into a given room with a hexagonal seating arrangementthan if one uses a more conventional square grid for the seating layout. Differently put, in the case of $d = 6$ ft, each audience member occupies 36 sq. ft in the square grid arrangement, but only 31.2 sq. ft in the hexagonal layout. This means that in very large spaces, a hexagonal layout will always accommodate the largest number of socially distanced seats.

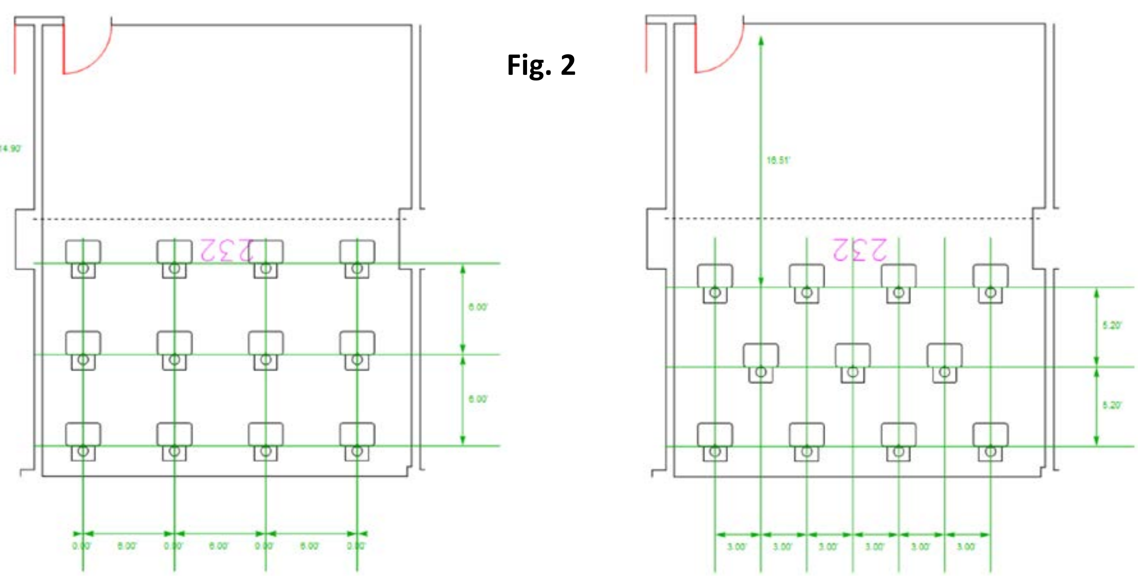
Fig. 1

Square grid
4 nearest neighbors
36 sq ft. (d^2) per student

Hexagonal grid
6 nearest neighbors
31.2 sq ft. ($\frac{2}{\sqrt{3}}d^2$) per student

But most spaces, in particular most classrooms, are not large. The exact dimensions of the space then has a large effect on what arrangements of seats allow for the largest number of seats. In the figure below a square and a hexagonal arrangement of desks have been added to a room with a 24’2”×29’8” footprint, with the constraint that no one’s face can be closer to the front of the room than 12’. Here the square layout actually beats out the hexagonal layout 12 seats to 11.

Fig. 2



Square grid – 12 seat capacity

Hexagonal grid – 11 seat capacity

As it turns out, neither of these arrangements is optimal. The biggest issue is that the full width of the room is not being used. There is no need to be socially distant from walls, so to avoid wasted space, seats should ideally be placed as close to the walls as possible, but with predetermined 6 ft sideways spacing between seats, this is feasible only if the width of the room (minus the width of one desk) is slightly larger than a whole multiple of 3 ft (or $d/2$), which is not true in this or most other cases. The simplest way to make use of the width of the room is to simply stretch the layout sideways until it become the same width as room. In the case of the hexagonal grid (but not the square grid), the stretching increases the distance between all the seats, which leaves room for compressing the layout in the lengthwise direction until the nearest neighbor distance again becomes 6 ft. In the case of the classroom of our example, the compression allows for a fourth row of seats to be added to the allotted space, raising the room capacity from 11 to 14. This arrangement (known as an orthorhombic grid) is then clearly superior to both the hexagonal and the square layouts.

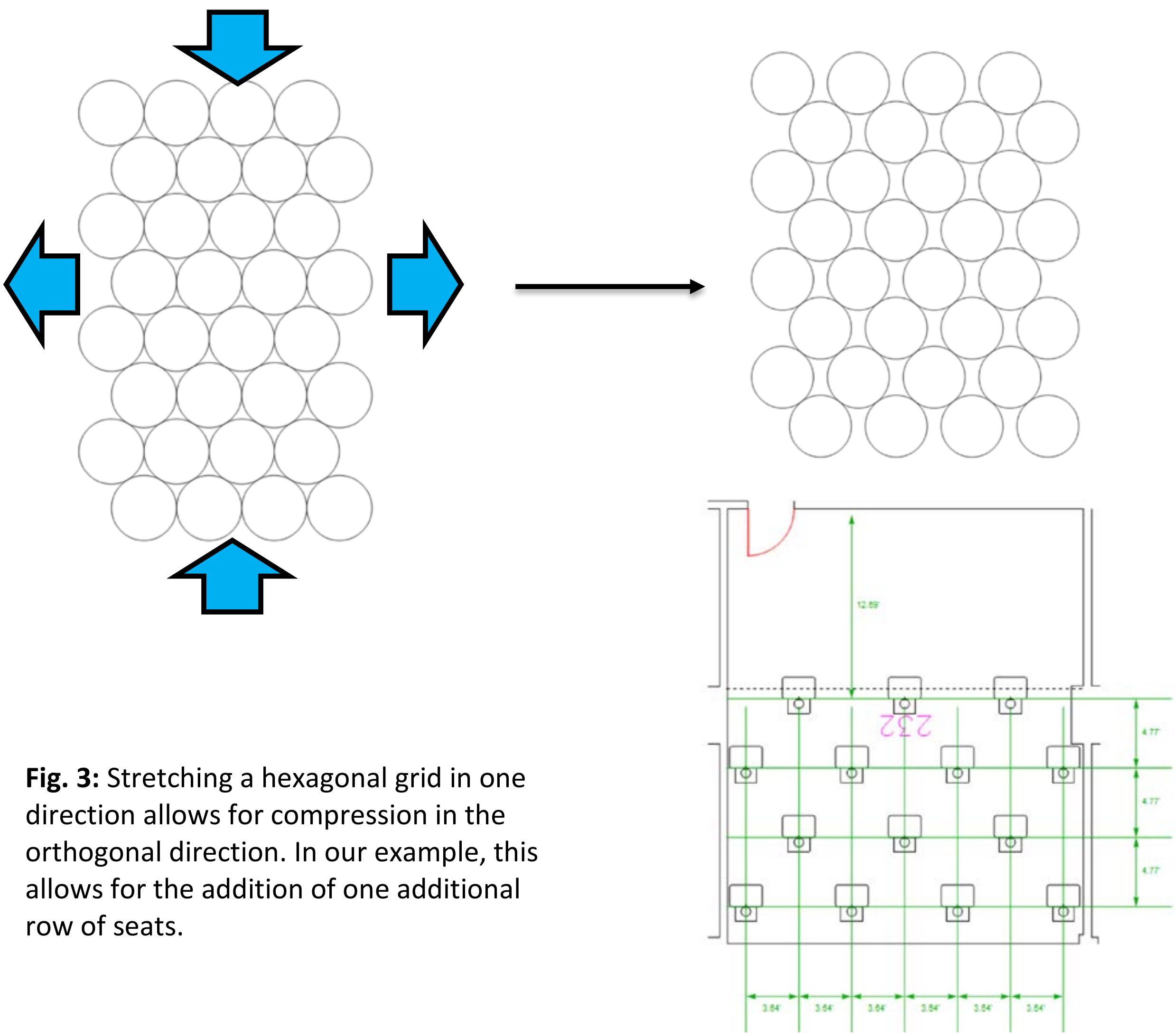


Fig. 3: Stretching a hexagonal grid in one direction allows for compression in the orthogonal direction. In our example, this allows for the addition of one additional row of seats.

The improvement in seating capacity enabled by stretching the layout is quite encouraging, so it seems a good idea to take this concept further. So far, the stretch in the horizontal direction did not change the number of lines of desks, but if we continue to stretch the layout further, fewer seats will fit in the horizonal direction. There is a chance that this loss of seats is offset by additional rows of seat that can be added to the layout, and if the space allotted for seats is sufficiently wide compared to its length, this may lead a further increase in seating capacity of the room.

As we continue to stretch the layout, the lengthwise distance between seats decreases until it reaches d. At this point, we have recovered a hexagonal grid, albeit one that is rotated by 90 degrees from our starting point. This distortion from hexagonal grid to hexagonal grid by way of orthorhombic configurations (as well as a square grid at the midpoint) is schematically illustrated in the upper portion of Fig. 4. It will contain most of the seating arrangements that may be of interest. However, there is an alternative distortion that takes us from one hexagonal configuration to the other that may be advantageous in some circumstances. It is illustrated by the lower portion of Fig. 4.

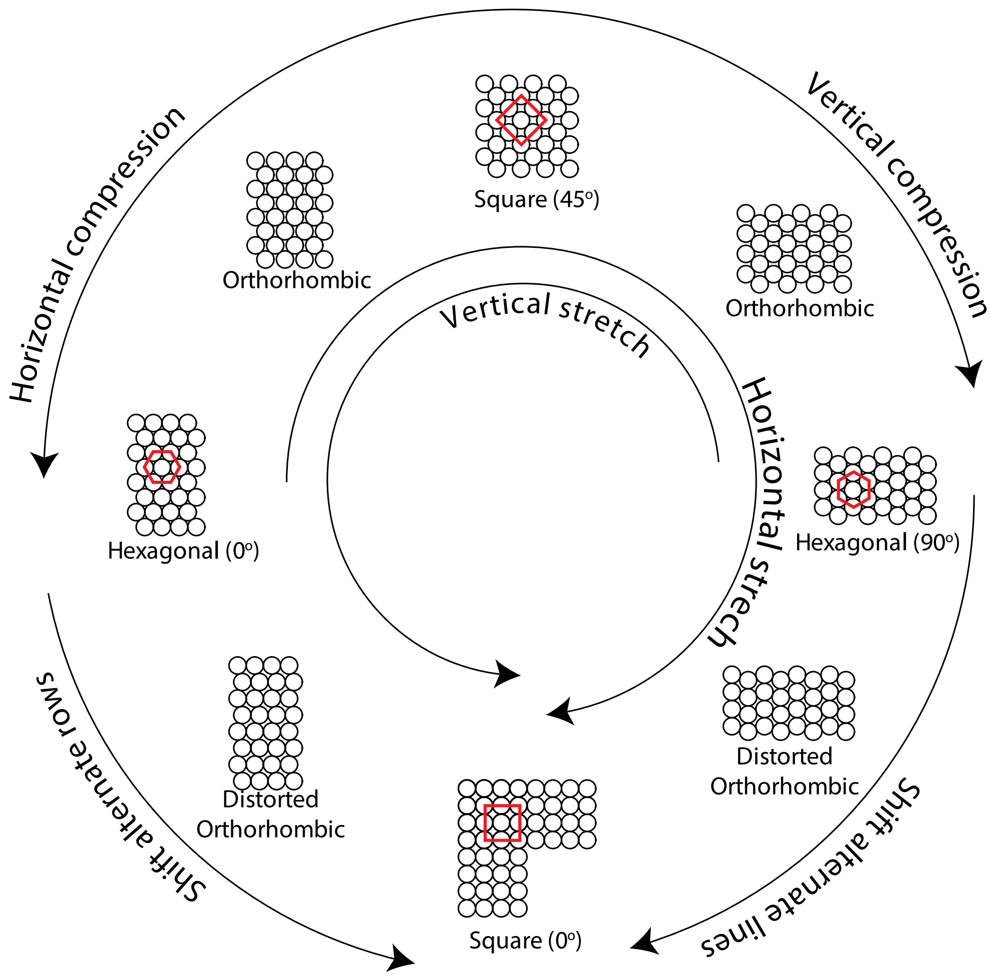


Fig. 4: Grid transformations explored by the tool

To obtain this distortion, we start with the rotated hexagonal grid and continue to stretch it in the horizontal direction. Since the lengthwise distance between seats is already at a minimum, the rows cannot be brought any closer together, but we can still maintain the condition that each seat is separated from four of its neighbors by 6 ft by shifting every other line of desks with respect to the other lines. If the conditions are right, this may add one additional row of seats to the layout. In our chosen example, the optimum configuration turns out to fall in this category. It allows for 15 seats in the classroom, one more than the stretched hexagonal grid above, even though it is not able to avoid the pillar on the right side of the room, and therefore is half a foot narrower. This layout is characterized by the fact that the distance between adjacent rows alternates between two different values, which comes about because the grid is a distortion of an orthorhombic grid. This feature will be present in all layouts found along the lower branch of Fig. 4.

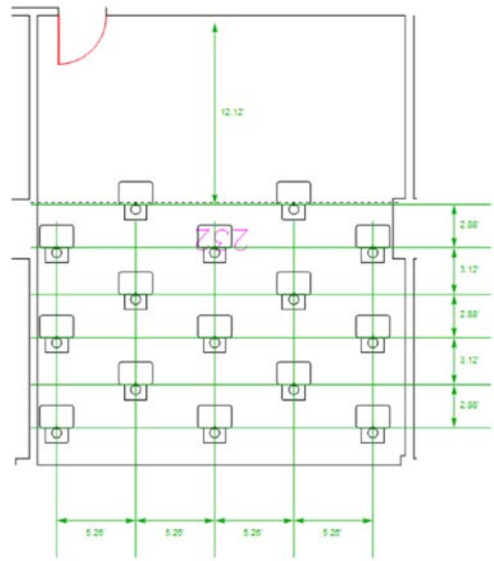


Fig. 5: Distorted orthorhombic grid – 15 seat capacity

The stretching of the grid can continue beyond this point until the sideways distance between desks reaches 12 ft (or $2d$), at which point every other line of desks will have shifted a full row's width, and one recovers a square layout like the one in Fig. 1(a). It is also possible to reach this square grid starting from the original hexagonal grid, by stretching it in the lengthwise direction. Since the seats in this arrangement are already separated by 6 ft in the sideways direction, we cannot reduce the distance between lines, but we can shift every other row to make sure that each disk retains contact with four others in the layout, we can continue until we again recover a square grid, closing the circle in Fig. 4.

A simple Mathematica program (gridstretch.nb or gridstretch.cdf) is available to visually illustrate these transformations.

Instructions

Getting started

When the Mathematica notebook is first opened, it will initialize automatically. As it initializes, it may look corrupted for a few seconds, but should then recover to the appearance in Fig. 6. If this does not happen, the tool can be reinitialized by selecting 'Evaluation>Evaluate Notebook' from the top menu.

Dynamic updating must also be enabled, which may require accepting a prompt on startup. Dynamic updating can also be enabled by ensuring that 'Evaluation>Dynamic Updating Enabled' is checked.

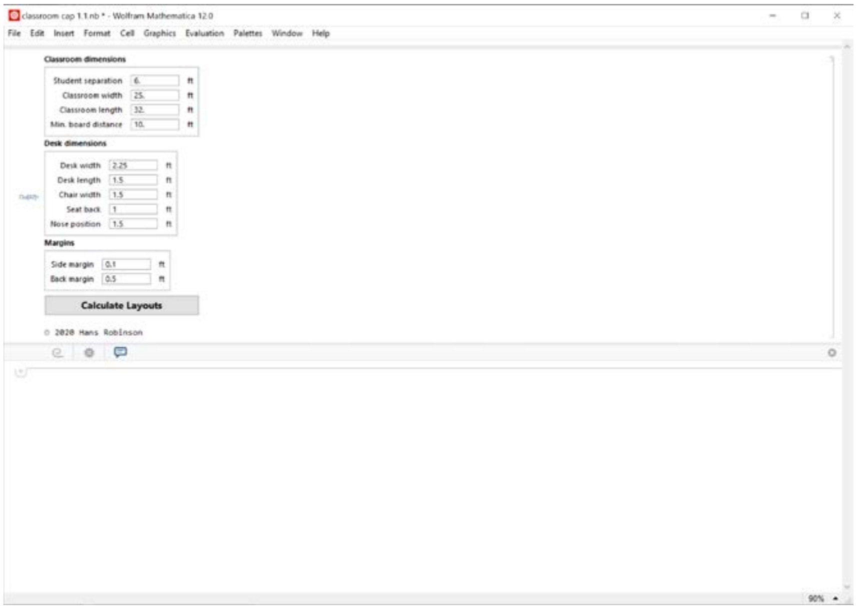


Fig. 6: Startup screen

Input dimensions

The first action should be to input the dimension of the space (classroom) and of the desks being used. All dimensions are currently assumed to be in feet (although a metric/imperial toggle is planned). For the classroom dimensions, give the length and width of the space, as well as the closest allowable distance to the front of the room (min. board distance) as needed either to give sufficient space for the instructor or presenter to maintain social distancing to the audience, or for the audience to be able to view any presentation with ease.

The meaning of the desk dimensions and margins is provided in Fig. 7. Separation between audience members is determined on a “nose-to-nose” basis, so the Nose distance dimension (measured from the front of the desk) is important. In particular, the distance to the front of the room (Board distance) is calculated based on this position, so it determines how many rows of desks can be placed in the room.

If the audience is seated on chairs without desks or tables, the Desk length should be set to 0, and the Desk width should be no larger than the Chair width.

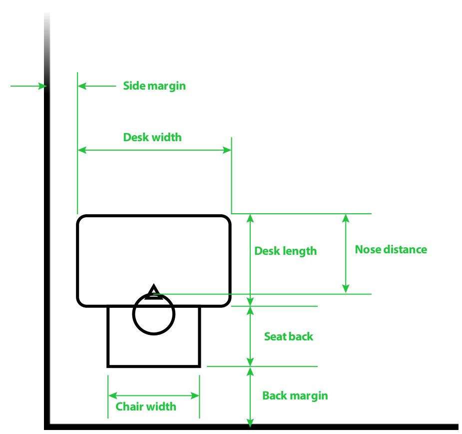


Fig. 7: Desk dimensions

Calculated layouts

Press the Calculate Layouts button to obtain a list of candidates for optimum classroom layout, which are listed in the table that appears below the button. The layouts are found using the transformation described above in the 'Principles of operation' section, and all layouts that make use of the full width of the space, consistent with the given desk dimensions and margins, are listed in the table. A drawing of the currently selected layout appears in the middle column of the screen, as shown in Fig. 8.

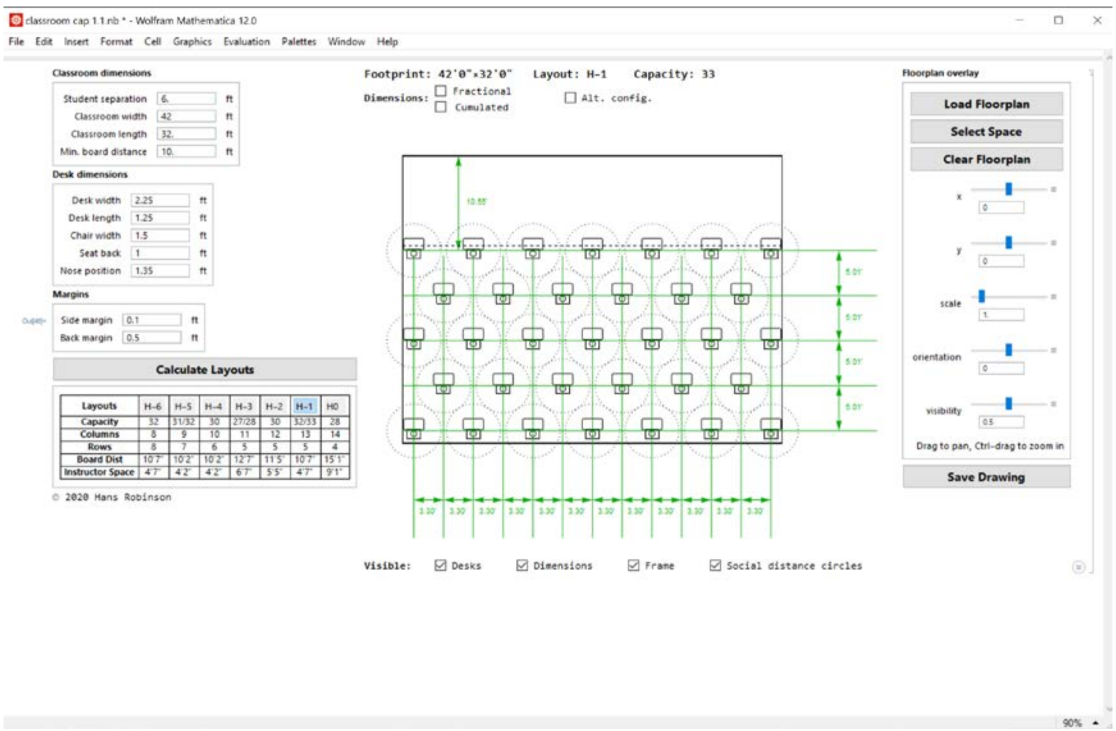


Fig. 8: Tool appearance after a layout has been calculated

The top row of the table lists the layout designation. Layout H0 has the same number of lengthwise lines of desks as the 0° Hexagonal layout (left side of the diagram in Fig. 4). In the example shown in Fig. 8, H0 has 14 lines of desks. As the layouts are stretched horizontally (corresponding to clockwise motion around the Fig. 4 diagram, lines are removed from layout, which is reflected in the layout designation. For example, layout H-2 has two fewer lines than H0 (12 in Fig. 8). If the number of lines in H0 is odd, a layout H1 may also be generated, corresponding to the distorted orthorhombic layout in the lower left quadrant of Fig. 4. If any layouts from the lower right quadrant of Fig. 4 are of interest, these will appear on the left side of the table.

In addition to the layout designation, the table also lists seating capacity for each layout, along with the number of rows and lines of desks in the layout, the distance of the frontmost audience members to the front of the room, and the space available for any socially distanced presenter at the front of the room.

Layout drawing

Click on the layout designation in the table to generate a drawing of that layout in the middle column of tool. The room dimensions, layout name, and seating capacity of each layout appears as a header at the top of this column. To facilitate the implementation of the layout in a physical room, checking the 'fractional' box in this area changes the drawing dimensions from decimal feet to feet and inches. Similarly, the 'cumulated' checkbox toggles dimensions from standard to cumulated form.

In addition, each layout exists on two separate forms, and the 'Alt. config." checkbox toggles between these. If the number of lines and rows in the layout are both odd, the alternate configuration holds one fewer seats than the default configuration, but in return avoids placing desks in room corners where pillars and other obstruction may exists, and also prevents audience members in the front from sitting at the margins of each row, where they may be viewing the front of the room at an inconveniently large angle.

In the figure, the horizontal dashed line indicates the minimum distance to the front of the room. The visibility of the other elements of the drawing, including the frame that outlines the room footprint, the dimensions, social distance circles, and the desks themselves, can be toggled with the checkboxes immediately below the drawing.

Floorplan overlay

Note: Due to license limitations imposed by Wolfram, the functionalities in this section are not implemented in the CDF version of the tool. This means that it is not possible to overlay the desk layouts on floorplans, nor can the layouts be saved to a pdf file. The algorithm that calculates the different optimized layouts is however the same in both versions, and the drawings can always be saved using screen capture, so the core functionality of the tool is preserved in the CDF version.

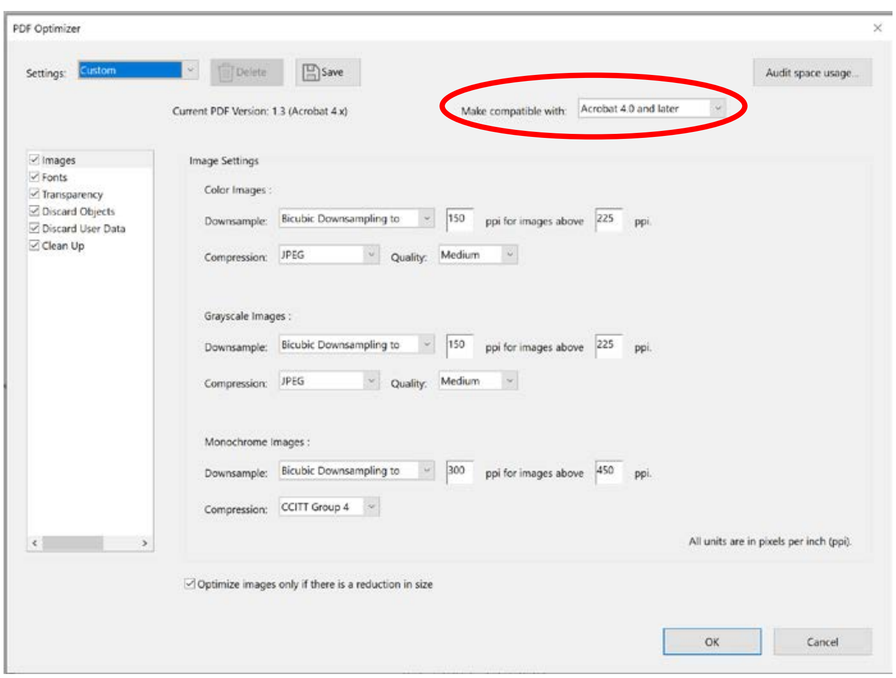


Fig. 9: Selection in Adobe Acrobat to resave PDF files in a Mathematica-compatible format.

<u>Note:</u> The ability of Mathematica to parse newer version of the PDF format is limited. Therefore, it is highly recommended that any floorplans be converted to PDF version 1.3 (Acrobat 4.x). This can be done with different software tools. For example, in Adobe Acrobat, select 'File>Save as Other>Optimized PDF…", and choose "Acrobat 4.0 and later" in the "Make compatible with:" drop-down list, and then click "OK" to save. Since floorplans generally are simple line drawings, this should result in no loss of functionality.

Floor plans can be loaded from PDF plans in two step process. First the full pdf file is loaded by clicking on the 'Load Floorplan' button. Second, the space under consideration is chosen in the dialog accessed through the 'Select Space' button. This ensures that the graphics used for the overlay is not so complex it slows down the tool. The procedure for selecting a specific space is as follows:

1. Double click on the floor plan at the top of the popup window that appears when you click on the 'Select Space' button. This should turn the frame around the drawing from orange to gray.
2. Click and drag on the plan to select the space for which seating should be optimized.
3. Right-click on the marked graphics and select 'Copy Graphics Selection'
4. Place the cursor in the smaller space in the lower left of the window, and paste (Ctrl-V or right-click>Paste)
5. Click 'OK'. The window will close and the plan of the selected space should appear superimposed on the calculated layout.

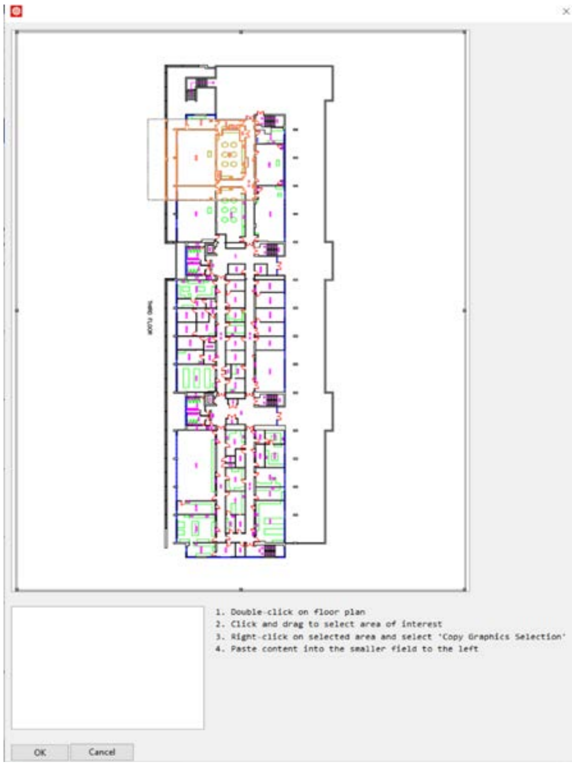
Note that it may take a few seconds before the selection window appears after the button is pressed.

This procedure is clearly more cumbersome then needed, and if possible, it will be replaced in a later version. Import of file formats other than PDF may also be added on request.

The next step is to register the overlaid graphics to the calculated layout. The controls that appear below the 'Clear Floorplan' button can be used to do this. Here, *x* and *y* translate the overlay, *scale* zooms the image, and *orientation* rotates it. The *scale* control causes zooming to occur centered on the lower left corner of the frame, so it is likely best to first align the layout with this point, and zoom until the room matches the frame.

Alternatively, and perhaps more easily, the layout can be translated by clicking and dragging in the layout graphic, and zooming in can be done with Ctrl-Click and drag. It is advantageous to deselect the visibility of the desks and dimensions during the zooming process, and then deselect the frame before saving the image, as show below in Figure 11.

Finally, the visibility control can be used to regulate the visibility of the overlay. The default value is a 50% reduction in visibility.



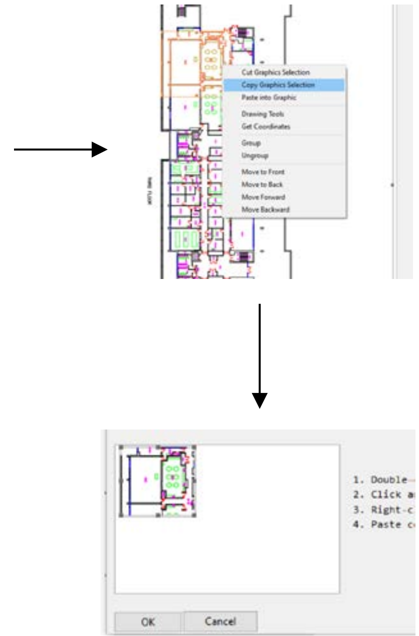
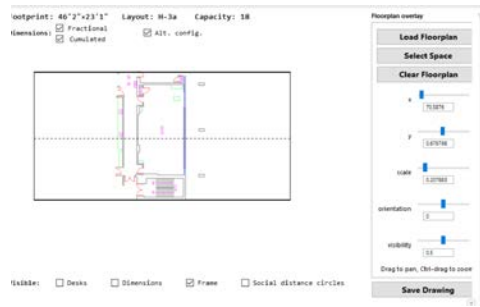


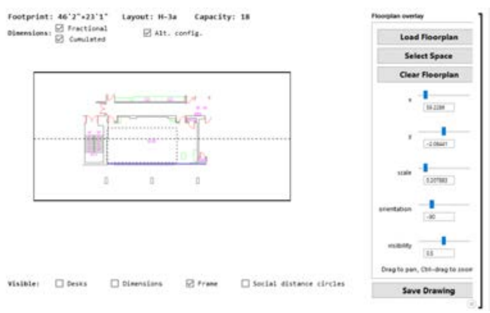
Fig. 10: Procedure for selecting overlay graphics

Once the drawing has been satisfactorily adjusted, it can be saved as a PDF file by clicking on the ‘Save Drawing’ button. On request, additional file formats may be added.

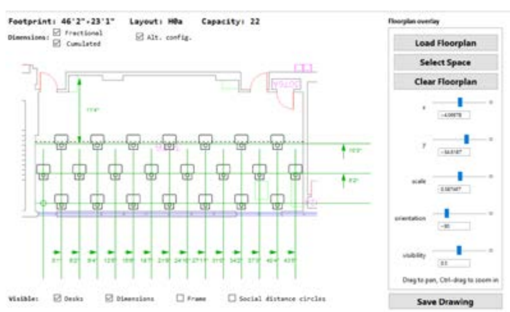
At this point, it is possible select another space from the same floor plan by again clicking on the ‘Select Space’ button, or else a new plan can be loaded with the ‘Load Floorplan button’.



Overlay as loaded, with desk visibility disabled



Zooming in with mouse controls



Resulting drawing with desk visibility reenabled, but frame visibility disabled

Fig. 11: Overlaying floorplan with calculated layout